

MATH Math 162A Review: Einstein Convention.

1. Prove the Cauchy inequality

$$\left(\sum_{i=1}^n x_i y_i\right)^2 \leq \sum_{i=1}^n x_i^2 \cdot \sum_{i=1}^n y_i^2$$

using the Einstein convention.

Solution: We discussed the inequality in the previous video. Here we use the Einstein convention to give a "better" proof.

The Left hand side, by the Einstein Convention can be written as

$$\left(\sum_{i=1}^n x_i y_i\right)^2 = \sum_{i=1}^n x_i y_i \sum_{j=1}^n x_j y_j = x_i y_i x_j y_j$$

The right hand side, is

$$\sum_{i=1}^n x_i^2 \sum_{i=1}^n y_i^2 = \sum_{i=1}^n x_i^2 \sum_{j=1}^n y_j^2 = x_i^2 y_j^2$$

$$\begin{aligned} \text{LHS} - \text{RHS} &= x_i^2 y_j^2 - x_i x_j y_i y_j \\ &= \frac{1}{2} (x_i y_j - x_j y_i)^2 \geq 0 \end{aligned}$$